Maths Academy, 4th Nov 2013: Ellipses

Draw a **circle**. Mark its centre and its radius.	Draw an **ellipse** (oval). Mark the centre, and its *minor* and *major* axes.

Equation of a circle. Draw a circle of radius *r* centred on (0,0). Now mark a point on the circle with coordinates (*x,y*). Use **Pythagoras theorem** to write down an equation relating *x, y* and *r*.

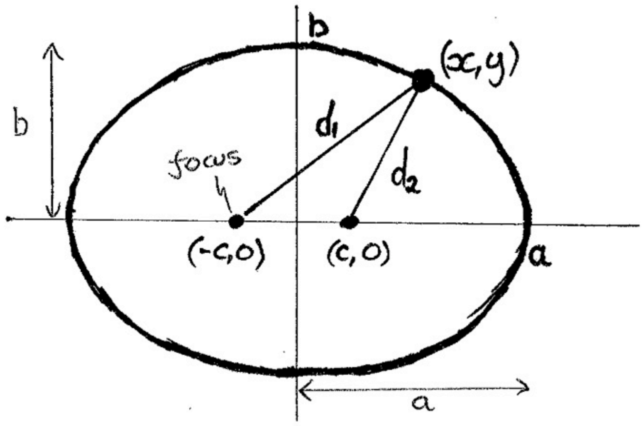
Show that your equation can be rewritten like this: $$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Stretch a circle. We may stretch a circle in (e.g.) the *x* direction by changing the "radius" in the denominator. Try sketching these two shapes: $$\text{(a)}\ \frac{x^2}{4r^2} + \frac{y^2}{r^2} = 1, \qquad \text{(b)}\ \frac{4x^2}{r^2} + \frac{y^2}{r^2} = 1$$

Sketch the stretched circle defined by this equation (assume *a*>*b*):
What is the meaning of *a* and *b*? $$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Definition: Take a string and attach its ends to two fixed points. Pull the string taut somewhere in the middle. Move the middle point around, keeping the string taut – this traces out an ellipse.

Mathematically, an ellipse is the set of all points *P* such that $d_1 + d_2 = l$ is a constant, where d_1 & d_2 are the distances from *P* to the fixed points, and *l* is the length of the string. Each fixed point is called a **focus** of the ellipse.



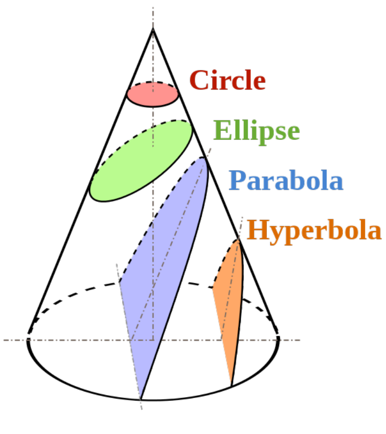
In this picture, the foci at (-c,0) and (c,0) are joined by a string of fixed length $l > 2c$.

1. Use Pythagoras theorem to write down equations for d_1 & d_2 in terms of *x, y* and *c*.

2. By considering two special points on the ellipse, show that $a = l/2$, and $b^2 = a^2 - c^2$, where *a* and *b* are the constants in the Cartesian equation of an ellipse:
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

3. (Difficult!) Prove that $d_1 + d_2 = l$ is equivalent to the Cartesian equation above.

Conic Sections: Imagine taking a "slice" through a cone – what would you find? A *conic section* is any curve found in this way. Ellipses and circles are two examples of conic sections. Two other examples are parabolas and hyperbolas.



Sketch a parabola and a hyperbola.

Sketch the solution of $$\frac{x^2}{4} - \frac{y^2}{9} = 1$$