

# RESEARCH STATEMENT

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## 1. DEHN SURGERY

Dehn surgery on a link  $L$  in a 3-manifold  $M$  is the process of gluing solid tori to the boundary components of  $M$  minus an open regular neighbourhood of  $L$ .

1.1. **Recent.** We will write 5CL to denote the minimally twisted 5-chain link (see Figure 1). Most manifolds in the Callahan - Hildebrand - Weeks classification of hyperbolic manifolds [CHW] of small volume are described by surgery on 5CL [DT]. Moreover, many well known classes of manifolds with “interesting” fillings are obtained by surgery on 5CL (see [Bak] [Kan], [EM]). My Ph.D. thesis [Ro1] gives a complete classification of the exceptional surgeries (i.e. non-hyperbolic) on 5CL.

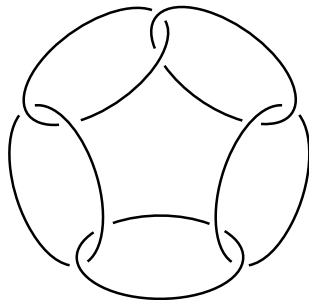


FIGURE 1. The minimally twisted chain link with 5 components.

The results in [Ro1] are dependent on numerical approximations to solutions of Thurston’s hyperbolicity equations obtained using Snappy [CDW]. In joint work with Carlo Petronio and Bruno Martelli [MPR], we used results of Moser [Mos] to make the results in [Ro1] independent of numerical approximations.

The classification in [MPR] gives an indirect classification of the exceptional slopes and fillings of *all* manifolds obtained by surgery on 5CL; I organised this classification in [Ro2] to provide experimental results that support conjectures made by Acuña, González, Gordon, and Short listed in [KL].

1.2. **Current.** While much is understood about exceptional pairs (distinct non-hyperbolic surgeries) on hyperbolic links in an arbitrary 3-manifold, see [MO] for a survey, much less is known about hyperbolic knots in  $S^3$ . In ongoing work with Benjamin Audoux, Ana Garcia Lecuona, and Daniel Matignon I am in the process of describing explicit knot diagrams of all hyperbolic knots in  $S^3$  obtained by surgery on 5CL with exceptional pairs at maximal known distance.

## 2. OTHER INTERESTS IN KNOT THEORY AND LOW DIMENSIONAL TOPOLOGY

I have a broad interest in knot theory and low dimensional topology. Over the coming year, I hope to continue my work on Dehn surgery and learn about knot homology theories, L-spaces, and geometric group theory.

2.1. **The past.** As an M.A.St. student I learned about almost normal surfaces and the 3-sphere recognition problem by working through [Tho] and as an M.Sc. student I worked on problems about finite type invariants of knots; this work contributed to [Ro3] and [BHLR].

2.2. **The future.** To further my research program, I will continue to work on problems from Dehn surgery and develop my collaboration with Ana Garcia Lecuona and Benjamin Audoux; we have plans to work on problems connected to quasi alternating Montesinos links, Khovanov homology and error correcting quantum codes. Among other projects, I highlight the following problems:

**More Dehn surgery.** In [Bak], Baker shows that every Berge knot is obtained by surgery on some chain link, and that “most” Berge knots are obtained by surgery on 5CL. Using the results in [Ro1], enumerating *every* knot with a lens space surgery obtained by surgery on 5CL becomes tractable. A natural problem is to show that every example in this enumeration is a Berge knot. My current work with Audoux, Garcia Lecuona, and Matignon enumerates all hyperbolic knots with two lens space surgeries; given the results in [Bak], it is worth trying to prove that these are the *only* Berge knots, and hence conjecturally the only hyperbolic knots in  $S^3$ , with two lens space surgeries.

**Quasi-alternating Montesinos links.** One of the current central themes of low dimensional topology is Heegaard-Floer homology, introduced by Ozsváth and Szabó in [OS1] and [OS2]. The 3-manifolds with the simplest Heegaard-Floer homology are called L-spaces and they are not yet fully understood. It is well known that the double cover of  $S^3$  branched over a quasi-alternating link is an L-space [OS3]. The explicit enumeration of all quasi-alternating Montesinos links is relevant to conjectures made in [CO] and [QCQ], and can be considered as a first step in the classification of Seifert-fibered L-spaces, complementing the results in [LS]. Ana Garcia Lecuona and I intend to look at the problem of enumerating quasi-alternating Montesinos links.

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