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Homework Q4, Q8, Q 24 for week 3 tutorial

From last time:

$(-1).\underline{v} = -\underline{v}$

Proof: $\underline{0} \overset{\text{prop } 1.2a}{=} 0\underline{v} = (1-1)\underline{v}$

$\overset{VS4}{=} 1.\underline{v} + (-1).\underline{v} \overset{VS2}{=} \underline{v} + (-1)\underline{v}$

Adding $-\underline{v}$ to LHS & RHS we get

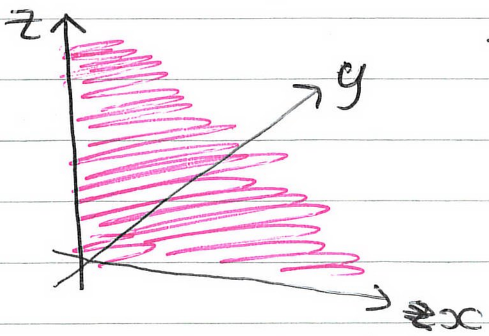
$\underline{0} - \underline{v} = -\underline{v} = \underline{v} + (-1)\underline{v} - \underline{v} \underset{\text{VS1 inv \& comm}}{=} (-1)\underline{v}$

Vector spaces

$C[0,1] = \{f:[0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$

$\mathcal{F}([0,1],\mathbb{R}) = \{f:[0,1] \to \mathbb{R}\}$ $\quad C \subseteq \mathcal{F}$

$\text{span}\{(1,0,0),(0,0,1)\} \subseteq \mathbb{R}^3 = \Pi$



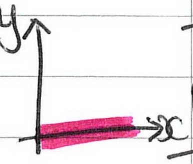
Π is a subspace of $\mathbb{R}^3$ (in the MAS201 sense)

* Π is closed under addition & scaling.
* $\underline{0} \in \Pi$

<u>Definition</u> Let V be a vector space over a field $\mathbb{F}$. A set $W \subseteq V$ is called a subspace of V if:

* $\underline{0} \in W$
* $\underline{u}+\underline{v} \in W$ for $\underline{u}, \underline{v} \in W$
* $c\underline{v} \in W$ for all $\underline{v} \in W$ & $c \in \mathbb{F}$

<u>Ex</u> $C[0,1]$ is a subspace of $\mathcal{F}([0,1], \mathbb{R})$



<u>Proof</u> $0(x):[0,1] \to \mathbb{R}$ $x \mapsto 0$ continuous

$\underline{0} \in C[0,1]$

see 207 for proof

if f & $g \in C[0,1]$ then $f+g \in C[0,1]$

If $c \in \mathbb{R}$ & $f \in C[0,1]$ then $cf \in C[0,1]$

So, $C[0,1]$ is a subspace of $\mathcal{F}([0,1], \mathbb{R})$

$A = \{f:[0,1] \to \mathbb{R} \mid f(1)=1\}$

$\underline{0} \notin A$ $\therefore$ not a vector subspace of $C[0,1]$

$C^\infty[0,1] := \{f:[0,1] \to \mathbb{R} \mid f^{(n)} \text{ exists } \forall n\}$

$\subseteq C[0,1]$

* $\underline{0} \in C^\infty[0,1]$ because $\frac{d^n}{dx^n}(0(x)) = 0(x)$

* If $f, g \in C^\infty[0,1]$ then

$$\frac{d^n}{dx^n}(f+g) = f^{(n)} + g^{(n)} \quad \text{so } f+g \in C^\infty[0,1]$$

* $c \in \mathbb{R}$, $f \in C^\infty[0,1]$ then $cf \in C^\infty[0,1]$

$$\frac{d^n}{dx^n}(cf) = cf^{(n)}$$

<u>Proposition</u> Let W be a subspace of a vector space V over a field $\mathbb{F}$.

a) If W is a subspace of V then W is a vector space over $\mathbb{F}$.

Proof: Verify $(W, +)$ is Abelian.

(VS2–VS5 hold in W because they hold in V)

* <u>Closure</u>: $\underline{w}_1 + \underline{w}_2 \in W$ when $\underline{w}_1, \underline{w}_2 \in W$
* $(W, +)$ is <u>associative</u> & <u>commutative</u> because ~~$W \in V$~~ they hold in $(V, +)$
* <u>Identity</u>: $\underline{0} \in W$ <u>because W is a subspace</u>
* <u>Inverse</u>: we know $c\underline{w} \in W$ $\forall c \in \mathbb{F}$ & $\underline{w} \in W$. Let $c = -1 \in \mathbb{F}$ we see $-\underline{w} \in W$

b) $U \subseteq V$ is a subspace iff
* $\underline{0} \in U$
* $\underline{u} + c\underline{v} \in U$ when $\underline{u}, \underline{v} \in U$ & $c \in \mathbb{F}$

Love
Love
Love x

Proof: $U \subseteq V$ Over $\mathbb{F}$

① $\underline{0} \in U$
② $c\underline{u} \in U$ for $\underline{u} \in U$, $c \in \mathbb{F}$
③ $\underline{u} + \underline{v} \in U$ for $\underline{u}, \underline{v} \in U$

(a) $\underline{0} \in U$
(b) $c \in \mathbb{F}$, $\underline{u}, \underline{v} \in U \Rightarrow \underline{u} + c\underline{v} \in U$

$a \Leftrightarrow b$ means $a \Rightarrow b$ & $b \Rightarrow a$

Assume (a) & (b),
$\Rightarrow$ ① holds because ① = (a)
MAS201 ~~$\Rightarrow$ ② $\underline{u} = \underline{0}$, $c\underline{v} \in U$~~
$\Rightarrow$ ② holds when $\underline{u} = \underline{0}$ in (b)
$\Rightarrow$ ③ holds when $c = 1$ in (b)

Assume ①, ②, ③
$\Rightarrow$ (a) holds because (a) = ①
$\Rightarrow$ (b) holds because:
$z = c\underline{u} \in U$ by ②
$\underline{v} + z \in U$ by ③
ie. $\underline{v} + c\underline{u} \in U$

c) If $\{U_i\}_{i \in I}$ is a collection of subspaces of V then $\cap_{i \in I} U_i$ is a subspace of V.

Proof: $\underline{0} \in U_i \ \forall i \Rightarrow \underline{0} \in \bigcap_{i \in I} U_i$

if $u, v \in \cap_{i \in I} U_i$ & $c \in \mathbb{F}$

ie. $\underline{v} \in U_i \ \forall i \Rightarrow c\underline{v} \in U_i \ \forall i$

because U_i is a vector space.

$\Rightarrow \underline{u} + c\underline{v} \in U_i \ \forall i$ because U_i is a VS
$\Rightarrow \underline{u} + c\underline{v} \in \cap_{i \in I} U_i$

d) If U & W are subspaces of V then

$$U+W := \{ \underline{u}+\underline{w} \mid \underline{u} \in U \ \& \ \underline{w} \in W \}$$

is a subspace of V