Maths Academy, University of Sheffield

Problems for Mathematicians and Programmers

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Problems

Beware: some of these questions have not yet been solved by any (wo)man or machine! There are two questions on this sheet whose solution would surely be rewarded with the Fields Medal. Can you spot them?

1. Find the sums (a) $1 + 2 + 3 + \ldots + 100$, and (b) $\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{100}$.
2. Is 987654321 a prime number?
3. What is the remainder when 135797531 is divided by 9?
4. $3^2 + 4^2 = 5^2$ is an example of a Pythagorean triple. Find another 6 'primitive' triples $a^2 + b^2 = c^2$ where a, b, c do not share a common factor.
5. The Fermat numbers are
$$F_n = 2^{(2^n)} + 1.$$
i.e. $F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257$, etc. Are *all* the Fermat numbers prime numbers?
6. A Mersenne prime is a prime number of the form
$$M_p = 2^p - 1.$$
where p is prime. Find the first 8 Mersenne primes.
7. Find the smallest number with 2014 divisors (where here the divisors include 1 and the number itself).
8. The Fibonacci sequence is defined by $f_k = f_{k-1} + f_{k-2}$, and $f_0 = 0$, $f_1 = 1$. So the first few numbers are $0, 1, 1, 2, 3, 5, 8, 13$, etc. Compute the first 20 Fibonacci numbers.
9. Find the value of the ratio f_{k+1}/f_k in the limit $k \to \infty$.
10. A Perfect Number is equal to the sum of its proper divisors (i.e. including 1 but excluding the number itself). For example, $6 = 1 + 2 + 3$ is perfect. Find two other perfect numbers.
11. Are there an infinite number of Perfect Numbers? Prove your answer.
12. How many square numbers feature the digits 0 to 9 once and once only?
13. Find 10 palindromic primes greater than 100.
14. Prove that every even number greater than 2 can be expressed as the sum of two prime numbers.
15. Assemble the nine squares of sides $1, 4, 7, 8, 9, 10, 14, 15, 18$ into a rectangle.