THE EIGHTH SOMAS UNDERGRADUATE MATHEMATICS TEAM CHALLENGE

Description.

- This is the eighth SoMaS undergraduate mathematics team challenge. The challenge is open to *all* undergraduates. The challenge is a take home team event that consists of a collection of interesting problems suggested by members of staff. These are challenging problems (some much more than others). You should feel proud if you can work out (part of) the solution to any of these problems.

Most importantly.

- This is for fun and you are welcome to tell us anything you like about the problems that you felt were interesting or fun. These observations can include, but are not restricted to, (partial) solutions, special cases, humorous remarks, and generalisations or variations of the problems. Submissions which make the judges smile will be appreciated :-)

To take part.

- Email Fionntan Roukema (f.roukema@sheffield.ac.uk) to register and submit solutions before 23:59 on Friday the 5th of November. Your email should include an (un)imaginative name for your team as well as a list of your team's members (which can be greater than or equal to one).

Other remarks.

- There will be a variety of awards available to people who enter. These will be given for the best team name, the most valuable solution to a problem, the overall best submission, as well as for honourable mentions.

2021-22 Problems

Problem 1: You are in a room, blindfolded, and either side of you there are two tables with pound coins laid out flat. You do not know how many coins there are on either table, but you are told that the table on your left shows exactly 25 heads while the table on your right shows exactly 7 heads. Your task is to devise a procedure involving moving coins from one table to the other so that both tables show an identical number of heads.

Problem 2: Start with the integers $1, 2, \ldots, 2021$. Choose two of the integers and replace them with their difference, producing a list of 2020 integers. Continue the process of replacing two numbers by their difference. After 2020 operations, we will end up with a single integer. Can this final integer be 0?

Problem 3: Write a Python programme which:

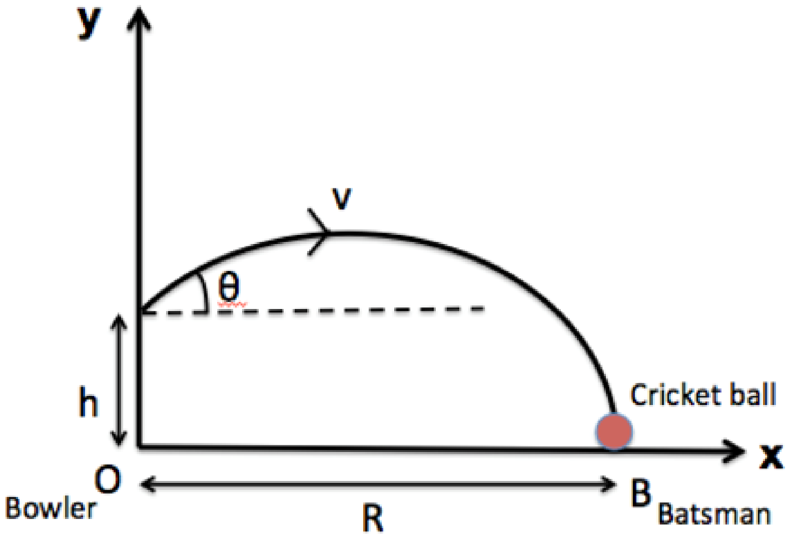
- is less than 256 characters long;
- would generate the longest string of nines that you can (and then stop).

You don't have to run it!

Problem 4: N coins are placed on an infinite chessboard so that each square is either empty or contains exactly one coin. Describe a procedure which only involves flipping coins, so that in the final configuration the difference $|\#\text{Heads} - \#\text{Tails}| \leq 1$ for each row, and for each column. You should prove that your procedure works for all N.

Problem 5: Let $A_1, A_2, \ldots, A_n$ be distinct subsets of $\{1, 2, \ldots, 2021\}$ such that any pairwise intersection $A_i \cap A_j$ (including the case $i = j$) contains an even number of elements. How large can n be?

Problem 6: Recently T20 men's cricket world cup competition started between 16 teams. It requires teams to score runs at a very fast rate as there are limited number of overs. In addition to a powerful batsman, a skilful bowler can also help the team by restricting the runs of the opponents. Some pitches can support fast bowling which can make it difficult for the opponents to score many runs. Another possibility is that the bowler is skilful enough to bowl a Yorker (a ball pitching near the batsman's feet) from time to time. One of the crucial things with the Yorker is the angle at which the ball is released from the bowler's hand. In order to get some idea, consider the figure as shown below:



Considering the constant gravity as the only force acting on the ball, derive the following equation

$$R = \frac{v\cos(\theta)\left(v\sin(\theta) + \sqrt{v^2\sin^2(\theta) + 2gh}\right)}{g}$$

where R is the distance between the bowler at O and the batsman at B; h is the height from where the bowl is released by the bowler. This equation can be used to deduce the theoretical optimum angle θ. For typical bowling speed, $v = 30\text{ms}^{-1}$ and acceleration due to to gravity, $g = 9.81\text{ms}^{-2}$ what realistic values of θ exist if $R = 20\text{m}$ and $h = 2\text{m}$? Give comments.