# THE SIXTH SOMAS UNDERGRADUATE MATHEMATICS CHALLENGE

Description. This is the sixth SoMaS undergraduate mathematics challenge. The challenge is open to *all* undergraduates. The challenge is a take home team event that consists of a collection of interesting problems suggested by members of staff. These are challenging problems (some much more than others). You should feel proud if you can work out (part of) the solution to any of these problems.

Above all, this is for fun and you are welcome to tell us anything you like about the problems that you felt were interesting or fun. These observations can include, but are not restricted to, (partial) solutions, special cases, humorous remarks, and generalisations or variations of the problems.
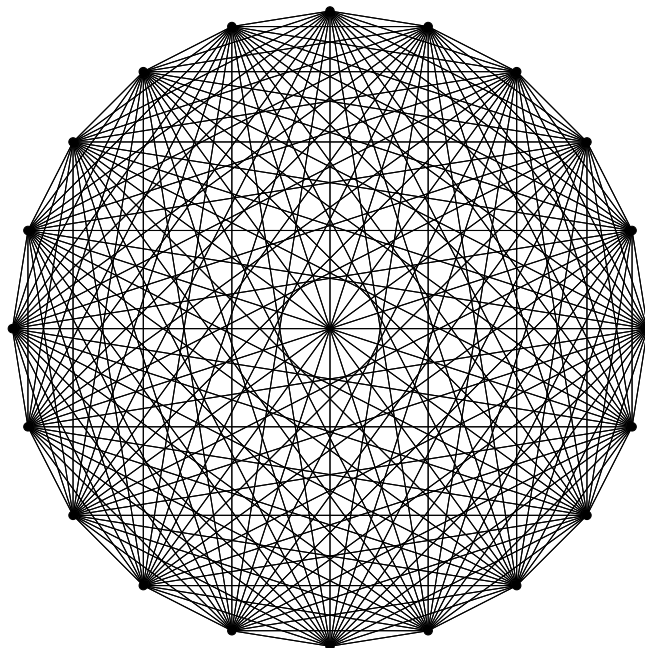
To take part.
- Email Fionntan Roukema (f.roukema@sheffield.ac.uk) to register and submit solutions before 23:59 on Friday the 2nd of November. Your email should include an (un)imaginative name for your team as well as a list of your team's members (which can be greater or equal to one).

Other remarks.
- There will be a variety of prizes and awards available to people who enter. These will be given for the best team name, the most valuable solution to a problem, the overall best submission, and for honourable mentions.

2018-19 Problems.

(1) In the film True Lies, Arnie is on a horse chasing a man on a motorbike. The man on the motorbike drives off the top of a skyscraper and lands in a swimming pool on the top of a building on the other side of the road. Arnie attempts the jump on his horse, but his horse balks at the last moment. See `https://www.youtube.com/watch?v=Z-GlKQ_tP18` for context. Can you construct a mathematical model of the problem, and using physical arguments, determine whether Arnie is right to be disappointed, or whether Arnie's horse made the right decision?

(2) How many ten digit numbers containing ten distinct digits are divisible by 11111? A computer isn't needed to answer this problem.

(3) 20 points are regularly spaced on a circle of radius one. Line segments are drawn between every pair of points (see the picture on the other side). What is the product of the lengths of the line segments?

(4) When you buy a ticket in the weekly SoMaS lottery, you have a very small (but positive) fixed probability p of winning. In order to stimulate excitement, the organisers have announced that each week a free ticket will be sent either to Taylor Swift or to Jay Z, with Ms Swift receiving the first one. Great publicity will be generated when one of these international musical celebrities wins it for the first time. Which of them should receive the next free tickets in weeks $2, 3, 4, ...$, in order to make the probability of each of them winning it for the first time as close to equal as possible?

(5) Can you write $\mathbb{R}$ as a disjoint union of uncountably many uncountable sets, and can you do so explicitly?

(6) Christmas is coming in Lapland, the sleigh bells are ringing and the smell of gingerbread wafts from every chimney. Santa has asked the elves to begin sorting the mountain of presents which must be delivered this year.

Meanwhile, aspirational middle managers have given the elves a series of time consuming, tangential tasks. The elves have been asked to sort a two-dimensional grid of black and white tiles into a picture of a Christmas present.

Santa has gone away on a seasonal holiday to Macau with a sack full of money. Can you help the Elves to sort the present before Santa returns and fires everyone?

The task: "Sorting the present"
The following picture is a random sample of a 100 × 100 grid made up of 7118 black tiles and 2882 white tiles.

The elves want to rearrange these tiles into the following picture of a Christmas present, which also contains 7118 black tiles and 2882 white tiles.

However, the elves are only allowed to swap pairs of horizontally or vertically neighbouring tiles.

Your task is to write an algorithm that carries out 50,000 of these swaps, and which makes the first picture become as similar as possible to the second.

Getting started: *You can download an IPython notebook, which is ready made to generate random starting configurations and swap tiles of your choice, from:*
`http://nicfreeman.staff.shef.ac.uk/puzzles/sort_the_present.zip`
To get you started, the notebook contains a simple example algorithm which produces results such as: