THE THIRD SOMAS UNDERGRADUATE MATHEMATICS CHALLENGE

Description. This is the third SoMaS undergraduate mathematics challenge. The competition is open to *all* undergraduates. The competition is a take home team event that consists of seven interesting problems suggested by members of staff. These are challenging problems (some much more than others). You should feel proud if you can work out (part of) the solution to any of these problems.

Above all, this is for fun and you are welcome to tell us anything you like about the problems that you felt were interesting. These observations can include, but are not restricted to, (partial) solutions, special cases, humourous remarks, and generalisations or variations of the problems.

To take part. register a team by emailing:

- James Cranch (J.D.Cranch@shef.ac.uk) or
- Madeleine Jotz Lean (M.Jotz-Lean@shef.ac.uk) or
- Jayanta Manoharmayum (J.Manoharmayum@shef.ac.uk) or
- Fionntan Roukema (f.roukema@shef.ac.uk).

Your email should include an imaginative name for your team as well as a list of your team's members (which can be greater or equal to one).

- You should submit your best solutions by midnight on Friday the 6th of November.

Other remarks.

- With the permission of the relevant team, the best solutions will be posted on the competition website (`http://roukema.staff.shef.ac.uk/somas_competition.html`).
- There will be a variety of prizes and awards available to people who enter.

2015-2016 Problems.

(1) For $N = 201$, we can reverse the digits to get the number $\overline{N} = 102$. We have $M = 201 - 102 = 099$ and $\overline{M} = 990$. This gives $M + \overline{M} = 1089$.

Now let $N = abc$ be a general three digit number with integers $a, b, c \in \{0, ..., 9\}$ and $a > c$, and let $\overline{N} = cba$. Show that the three digit number $M = def := N - \overline{N}$ with $d, e, f \in \{0, ..., 9\}$ has the property that $M + \overline{M} = def + fed = 1089$ and generalise the result to base n.

(2) 100 lemmings are placed on a levitating 10 metre long steel beam. Each lemming is facing one end of the beam. They all start walking simultaneously in the direction they are facing. If two lemmings bump into each other they both turn and walk in the opposite direction. Are all the lemmings guaranteed to fall off the beam if they all move at a constant speed of 10 metres per hour? Can you find a minimum/maximum number of lemmings that will be on the beam after t minutes?

(3) Write down an explicit bijection from the open interval

$$I = (0, 1) := \{x \in \mathbb{R} \, : \, 0 < x < 1\}$$

to the infinite union of closed intervals

$$\bigcup_{n=1}^{\infty} [2n, 2n + 1].$$

(4) What is the largest value of

$$\sin(x)\sin(\sqrt{5}y) + \sin(\sqrt{2}y)\sin(\sqrt{6}z) + \sin(\sqrt{3}z)\sin(\sqrt{7}x)$$

that you can find, for real values of x, y and z between 0 and 100?

(5) If Andy Murray wins 51% of his serves and 50% of his other shots during Wimbledon 2016, what is the probability that he will win the tournament?

(6) For which $n \in \mathbb{N}$ does there exist a regular n−gon in the plane with all vertices having integer coordinates?

(7) [Problem 11852 from the *American Mathematical Monthly*][1] For $n \in \mathbb{Z}^+$, let $\nu_n = k$ if 3^k divides n but 3^{k+1} does not. Let $x_1 = 2$, and for $n \geq 2$ let

$$x_n = 4\nu_n + 2 - \frac{2}{x_{n-1}}.$$

So that (x_n) begins $2, 1, 4, \frac{3}{2}, \frac{2}{3}, 3, \ldots$. Show that every positive rational number appears exactly once in the list (x_n).

[1] This is a live problem and you can submit a solution to the journal for publication if you can find one. The judges don't know an answer, but we'd like to see you show us one!