Question 1. For $N = 201$, we can reverse the digits to get the number $\overline{N} = 102$. We have $M = 201 - 102 = 099$ and $\overline{M} = 990$. This gives $M + \overline{M} = 1089$.

Solution. Consider $N = 100a + 10b + c$ with $a > c$, $\{a, c \in \{0, \ldots, 9\}$. Then $\bar{N}$ is $100c + 10b + a$ and

$$N - \bar{N} = 100(a - c) + (c - a).$$

We want to write this in base 10. We have $c - a < 0$ (more precisely $-9 \leq c - a \leq -1$) so we replace it by $10 + (c - a)$ and correct accordingly. This yields

$$N - \bar{N} = (a - c) \cdot 10^2 + (-1) \cdot 10^1 + (10 + (c - a)) \cdot 10^0.$$

Since the coefficient before 10^1 is now negative, we need to do a second correction of the same type and get

$$M = N - \bar{N} = (a - c - 1) \cdot 10^2 + (10 - 1) \cdot 10^1 + (10 + (c - a)) \cdot 10^0 = (a - c - 1) \cdot 10^2 + 9 \cdot 10^1 + (10 - a + c) \cdot 10^0.$$

Finally we get

$$M + \bar{M} = (a - c - 1 + 10 - a + c) \cdot 10^2 + 18 \cdot 10^1 + (a - c - 1 + 10 - a + c) \cdot 10^0 = 900 + 180 + 9 = 1089.$$

In base n, we do almost the same computation. Consider $N = a \cdot n^2 + b \cdot n^1 + c \cdot n^0$ with $a > c$, $a, c \in \{0, \ldots, n - 1\}$. Then $\bar{N}$ is $c \cdot n^2 + b \cdot n^1 + a \cdot n^0$ and

$$N - \bar{N} = (a - c) \cdot n^2 + (c - a) \cdot n^0.$$

We want to write this in base n. We have $c - a < 0$ (more precisely $-n + 1 \leq c - a \leq -1$) so we replace it by $n + (c - a)$ and correct accordingly This yields

$$N - \bar{N} = (a - c) \cdot n^2 + (-1) \cdot n^1 + (n + (c - a)) \cdot n^0.$$

Since the coefficient before n^1 is now negative, we need to do a second correction of the same type and get

$$M = N - \bar{N} = (a - c - 1) \cdot n^2 + (n - 1) \cdot n^1 + (n + (c - a)) \cdot n^0 = (a - c - 1) \cdot n^2 + (n - 1) \cdot n^1 + (n - a + c) \cdot 10^0.$$

Finally we get

$$M + \bar{M} = (a - c - 1 + n - a + c) \cdot n^2 + 2(n - 1) \cdot n^1 + (a - c - 1 + n - a + c) \cdot n^0 = (n - 1)(n + 1)^2.$$

As mentioned above, this can be written as

$$M + \bar{M} = 1 \cdot n^3 + 0 \cdot n^2 + (n - 2) \cdot n + (n - 1) \cdot n^0.$$