THE SECOND SOMAS UNDERGRADUATE MATHEMATICS CHALLENGE

Description. This is the second SoMaS undergraduate mathematics challenge. The competition is open to *all* undergraduates. The competition is a take home team event that consists of five interesting problems suggested by members of staff. These are difficult problems of an elementary nature. You should feel proud if you can work out (part of) the solution to any of these problems.

To take part. register a team by emailing:

- Jayanta Manoharmayum (J.Manoharmayum@shef.ac.uk) or
- Fionntan Roukema (f.roukema@shef.ac.uk).

Your email should include an imaginative name for your team as well as a list of your team's members (which can be greater or equal to one).

- You should submit your best solutions by the end of week 4 (Friday the 6th of March).

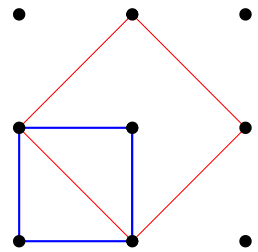
Other remarks.

- With the permission of the relevant team, the best solutions will be posted on the competition website (`http://roukema.staff.shef.ac.uk/somas_competition.html`).
- There will be a variety of prizes and awards available to people who enter.
- You are welcome to tell us about any observations related to the problems that you felt were interesting. These observations can include, but are not restricted to, generalisations or variations of the problems.

2014-2015 Problems.

(1) What is the largest prime that you can find with decimal expansion $2015\cdots$. Your answer should include a verification that your number is indeed prime.

(2) n^2 pegs are arranged in an $n \times n$ square grid. You have a piece of string which you can wrap around pegs of your choice. How many squares can you form? For example, if $n = 3$ then two such squares are shown below:



(3) A "cloning move" on an object at the point (n, m) moves the object to the point $(n, m+1)$ and puts a clone on $(n+1, m)$. Cloning moves are permitted only if $(n, m+1)$ and $(n+1, m)$ are unoccupied. If Dolly the sheep is standing at the origin and all other points are unoccupied, can she find a sequence of cloning moves that allow Dolly and her clones to escape from the set

$$\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\}?$$

(4) At the start of the 2013-14 Premier League football season, the Manchester United manager, David Moyes, complained about his teams early fixtures, saying, "I find it hard to believe that's the way the balls came out of the bag, that's for sure." He also said: "I think it's the hardest start for 20 years that Manchester United have had." (He later accepted the schedulers assurances that the fixtures were chosen randomly).

Manchester Uniteds first five fixtures were Swansea (A), Chelsea (H), Liverpool (A), Crystal Palace (H), Man City (A). In the 2012-13 season, Chelsea, Liverpool and Man City finished 3rd, 7th and 2nd respectively. Was there anything surprising about the difficulty of Manchester Uniteds first five fixtures?

(5) Show that there are arbitrarily large gaps between powers of integers. That is, show that for every $N \in \mathbb{N}$ there exits an $n \in \mathbb{N}$ such that no element of $\{n+1, \ldots, n+N\}$ is of the form m^k with $k > 1$.