THE FIRST SOMAS UNDERGRADUATE MATHEMATICS COMPETITION

Description. This is the first SoMaS undergraduate mathematics competition. The competition is open to *all* undergraduates. The competition is a take home team event that consists of five interesting problems suggested by members of staff. These are difficult problems of an elementary nature[1]. You should feel proud if you can work out (part of) the solution to any of these problems.

To take part. register a team by emailing:

- Jayanta Manoharmayum (J.Manoharmayum@shef.ac.uk) or

- Fionntan Roukema (f.roukema@shef.ac.uk).

Your email should include an imaginative name for your team as well as a list of your team's members (which can be greater or equal to one).

- You should submit your best solutions by the end of week 5 (Friday the 14th of March).

Other remarks.

- With the permission of the relevant team, the best solutions will be posted on the competition website (`http://roukema.staff.shef.ac.uk/somas_competition.html`).
- There will be a variety of prizes and awards available to people who enter.

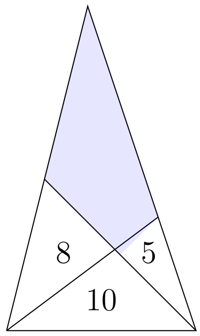
[1]the complete solution to one of this year's questions is an open problem in mathematics

2013-2014 Problems.

(1) Let $a_1 = 1$ and $a_{n+1} = (n+1)(a_n + 1)$. Find

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{a_n}\right).$$

(2) The following picture shows a (not to scale) triangle which is divided into four regions. The areas of three of the regions are shown. Find the area of the shaded region.



(3) Given an acute angled triangle T, show that there is a unique tetrahedron whose faces are all congruent to T and find the volume of this tetrahedron.

(4) For which $n \in \mathbb{N}$ can a square be divided into n (not necessarily congruent) squares?

(5) Define an n–*colouring* of the plane to be a function

$$f : \mathbb{R}^2 \to \{c_1, \ldots c_n\}.$$

In other words each point in the plane is coloured one of n colours. For which $n \in \mathbb{N}$ does there exist an n–colouring of the plane with the property that no two points in $\mathbb{R}^2$ at distance 1 apart have the same colour?